Multidimensional Mereotopology with Betweenness

Torsten Hahmann & Michael Grüninger
Semantic Technologies Laboratory,
Dept. of Computer Science & Dept. of Mechanical and Industrial Engineering,
University of Toronto
torsten@cs.toronto.edu, gruninger@mie.utoronto.ca

1. Objective: A Theory for Qualitative Spatial Maps

QUALITATIVELY abstract geometry to represent maps qualitatively while preserving essential aspects for route descriptions (without distances or absolute directions)

SEAMENTICALLY integrate theories of space:
mereologies, incidence structures, and geometries

MODULARLY DESIGN a hierarchy of first-order theories of qualitative space in a bottom-up fashion

2. Main idea: Abstraction of Geometry

ABSTRACT HILBERT’S ORDERED INCIDENCE GEOMETRY QUALITATIVELY SO THAT ONLY TOPOLOGY & ORDER MATTER (WITHOUT CURVATURE, SHAPE, OR DISTANCES).

We can describe city or building maps as sets of interconnected multidimensional features of uniform dimensions including aerial features (street blocks, administrative regions, parks, forests, water bodies), linear features (roads, rivers, rail lines), and point features (intersections, bridges, rail crossings, points of interest) using a standard geometry weakened by two common geometric assumptions:

(a) Two distinct entities A, B of equal dimension determine a unique C of next-highest dimension (line axiom);

(b) For two distinct entities A, B of equal dimension in a higher-dimensional space, there exists two more entities C, D of the same dimension so that C is in between A and B, and B is between A and D (continuity axiom).

3. Methodology: ‘Not Stronger Than Necessary’

COMPILATION OF THE MOST BASIC AXIOMS OF SPACE IN A WEAK THEORY AND AXIOMATIZE SUCCESSIVELY STRONGER EXTENSIONS INTERPRETED BY KNOWN SPATIAL THEORIES.

1. Show that the weak theory is a mereotope (i.e. the usual contact and parthood are definable): Construct theories stronger than existing mereologies without having to add essential mereotopological axioms

2. Show that suitable extensions are interpreted by known incidence structures and geometries (up to affine ~)

3. Extend the theory with multidimensional betweenness: a generalized version of geometric betweenness

4. Show that suitable extensions are interpreted by known ordered incidence geometries (such as ordered affine ~, betweenness ~, and Hilbert’s ~)

4. Weak Multidimensional Mereotopology

× Primitive relations:

SPATIAL CONTAINMENT Cont(x,y) (dimension-independent)

RELATIVE DIMENSION dim(x,y) = dim(x) − dim(y)

ZERO ENTITY ZEX(x)

× Relationship between containment and dimension:

Cont(x,y) → dim(x) ≤ dim(y) (CD-A1)

MODELING: Partial order defined by containment with superimposed linear order over equivalence classes of equidimensional entities (cf. Fig. 3)

× Definable relations:

CONTACT C(x,y) ⇒ Cont(x,y) ∧ Cont(y,x) (C-D)

PARthood P(x,y) ⇒ Cont(x,y) ∧ x ≠ min(y) (EP-D)

× Classification of contact into three types:

(PARTIAL OVERLAP) P(x,y) ⇒ x < y ∧ Cont(x,y) ∧ y = min(y)

INCIDENCE Inc(x,y) ⇒ x < y ∧ Cont(x,y) (or vice versa)

only a common entity that is part of one SUPERFICIAL CONTACT (C x y) ≥ Cont(x,y) ∧ y = min(y)

. . . contact without common part

Figure 1: Partial overlap, incidence, and superficial contact each in 2D and 3D (from left to right).

5. A “Sketch Map” of an Island and its Model in the Weak Multidimensional Mereotopology

Figure 2: Map with entities of various dimensions:

2D: ocean, main island, small island, city, lake;

1D: river main and arm, highway and central track;

0D: lighthouse main, lighthouse island.

6. Relationship to Other Mereotopologies, Incidence Structures, and Incidence Geometries

Figure 3: The model as partial defined by the containment relation amongst the primary spatial objects of Fig. 2.

Each bubble contains entities of identical dimension, partition is the containment relation within bubbles.

7. Multidimensional Betweenness

When is Betweenness Necessary?

Figure 5: Permuting Queen, Richmond, Adelaide, King, and Front St. does not change the map’s representation in the multidimensional mereotopology without betweenness.

Multidimensional Axiomatization of Betweenness

× Primitive relation: quaternary betweenness Btw(x,y,a,b) in M is strictly in between x and y

(B-A1) Btw(x,a,y,b) → a ≠ b ∧ b ≠ a

(B-A2) Btw(x,a,y,b) → Btw(x,b,a) (strongly reflexive)

(B-A3) Btw(x,a,y,b) → ¬Btw(x,a,b) (strictly acyclic)

(B-A4) Btw(x,a,y,b) ∧ Btw(y,a,b) → Btw(x,a,y) (outer transitivity)

(B-A5) Btw(x,a,b,y) ∧ Btw(x,a,y,b) → ¬Btw(x,a,y) (inner transitivity)

Figure 6: Intersecting streets, such as Dundas, Bloor, and The Queen’sway, can usually not be ordered. But Dundas, Annette, and Bloor might still be orderable.

7. Multidimensional Betweenness

When is Betweenness Necessary?

Figure 5: Permuting Queen, Richmond, Adelaide, King, and Front St. does not change the map’s representation in the multidimensional mereotopology without betweenness.

Multidimensional Axiomatization of Betweenness

× Primitive relation: quaternary betweenness Btw(x,y,a,b) in M is strictly in between x and y

(B-A1) Btw(x,a,y,b) → a ≠ b ∧ b ≠ a

(B-A2) Btw(x,a,y,b) → Btw(x,b,a) (strongly reflexive)

(B-A3) Btw(x,a,y,b) → ¬Btw(x,a,b) (strictly acyclic)

(B-A4) Btw(x,a,y,b) ∧ Btw(y,a,b) → Btw(x,a,y) (outer transitivity)

(B-A5) Btw(x,a,b,y) ∧ Btw(x,a,y,b) → ¬Btw(x,a,y) (inner transitivity)

Figure 6: Intersecting streets, such as Dundas, Bloor, and The Queen’sway, can usually not be ordered. But Dundas, Annette, and Bloor might still be orderable.

Future Challenge: Disentangle and Axiomatize the Various Usages and Interpretations of Betweenness

SEPARATION: The Humber separates Royal York from Jane

ENCLOSURE: Dundas and Bloor enclose Annette

VARIUS STRENGTHS OF PARTIAL BETWEENNESS: Parkside between Humber River and Dundas? (ambiguous)

Jane between Humber River and Keele? (likely)

Jane between Humber River and Parkside? (unlikely)